Geometric Approximation Algorithms

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Contents

Preface

> Finally: It was stated at the outset, that this system would not be here, and at once, perfected. You cannot but plainly see that I have kept my word. But I now leave my cetological system standing thus unfinished, even as the great Cathedral of Cologne was left, with the crane still standing upon the top of the uncompleted tower. For small erections may be finished by their first architects; grand ones, true ones, ever leave the copestone to posterity. God keep me from ever completing anything. This whole book is but a draft – nay, but the draft of a draft. Oh, Time, Strength, Cash, and Patience!
>
> – Moby Dick, Herman Melville.

This book started as a collection of class notes on geometric approximation algorithms that was expanded to cover some additional topics. As the book title suggests, the target audience of this book is people interested in geometric approximation algorithms.

What is covered. The book describes some key techniques in geometric approximation algorithms. In addition, more traditional computational geometry techniques are described in detail (sampling, linear programming, etc.) as they are widely used in developing geometric approximation algorithms. The chapters are relatively independent and try to provide a concise introduction to their respective topics. In particular, certain topics are covered only to the extent needed to present specific results that are of interest. I also tried to provide detailed bibliographical notes at the end of each chapter.

Generally speaking, I tried to cover all the topics that I believe a person working on geometric approximation algorithms should at least know about. Naturally, the selection reflects my own personal taste and the topics I care about. While I tried to cover many of the basic techniques, the field of geometric approximation algorithms is too large (and grows too quickly) to be covered by a single book. For an exact list of the topics covered, see the table of contents.

Naturally, the field of geometric approximation algorithms is a subfield of both computational geometry and approximation algorithms. A more general treatment of approximation algorithms is provided by Williamson and Shmoys [**WS11**] and Vazirani [**Vaz01**]. As for computational geometry, a good introduction is provided by de Berg et al. [**dBCvKO08**].

What to cover? The material in this book is too much to cover in one semester. I have tried to explicitly point out in the text the parts that are more advanced and that can be skipped. In particular, the first 12 chapters of this book (skipping Chapter 7) provide (I hope) a reasonable introduction to modern techniques in computational geometry.

Intellectual merit. I have tried to do several things that I consider to be different than other texts on computational geometry:

(A) Unified several data-structures to use compressed quadtrees as the basic building block and in the process provided a detailed description of compressed quadtrees.

(B) Provided a more elaborate introduction to VC dimension, since I find this topic to be somewhat mysterious.
(C) Covered some worthy topics that are not part of traditional computational geometry (for example, locality sensitive hashing and metric space partitions).
(D) Embedded numerous color figures into the text to illustrate proofs and ideas.

Prerequisites. The text assumes minimal familiarity with some concepts in computational geometry including arrangements, Delaunay triangulations, Voronoi diagrams, point-location, etc. A reader unfamiliar with these concepts would probably benefit from skimming or reading the fine material available online on these topics (i.e., Wikipedia) as necessary. Tail inequalities (i.e., Chernoff's inequality) are described in detail in Chapter 27. Some specific prerequisites are discussed in Chapter 28.

Cross-references. For the convenience of the reader, text cross-references to theorems, lemmas, etc., often have a subscript giving the page location of the theorem, lemma, etc., being referenced. One would look like the following: Theorem 19.3_{p257}.

Acknowledgments. I had the benefit of interacting with numerous people during the work on this book. In particular, I would like to thank the students that took the class (based on earlier versions of this book) for their input, which helped in discovering numerous typos and errors in the manuscript. Furthermore, the content was greatly affected by numerous insightful discussions with Jeff Erickson and Edgar Ramos. Other people who provided comments and insights, or who answered nagging emails from me, for which I am thankful, include Bernard Chazelle, Chandra Chekuri, John Fischer, Samuel Hornus, Piotr Indyk, Mira Lee, Jirka Matoušek, and Manor Mendel.

I would especially like to thank Benjamin Raichel for painstakingly reading the text and pointing out numerous errors and typos and for giving guidance on what needed improvement. His work has significantly improved the quality of the text.

I am sure that there are other people who have contributed to this work, whom I have forgotten to mention – they have my thanks and apologies.

A significant portion of the work on this book was done during my sabbatical (taken during 2006/2007). I thank the people that hosted me during the sabbatical for their hospitality and help. Specifically, I would like to thank Lars Arge (Aarhus, Denmark), Sandeep Sen (IIT, New Delhi, India), and Otfried Cheong (KAIST, Daejeon, South Korea).

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Errors. There are without doubt errors and mistakes in the text and I would like to know about them. Please email me about any of them that you find.

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