University of Illinois at Urbana-Champaign
Department of Computer Science

Midterm 2
CS 273 Introduction to Theoretical Computer Science
Spring 2004

Name:
Netid:

- Print your name and netid, neatly in the space provided above; print your name at the upper right corner of every page. Please print legibly.

- This is a closed book exam. No notes, books, dictionaries, pans, or calculators are permitted.

- Write your answers in the space provided for the corresponding problem. Let us know if you need more paper.

- Suggestions: Read through the entire exam first before starting work. Do not spend too much time on any single problem. If you get stuck, move on to something else and come back later.

- If you run short on time, remember that partial credit will be given.

- If any question is unclear, ask one of us for clarification.

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1. **Graphs**  
   **[20 Points]**
   Prove that if a graph \( G \) is self complementary (isomorphic to its complement) then the number of vertices in \( G \) is either divisible by 4, or leaves a remainder of 1 when divided by 4.

2. **Trees**  
   **[20 Points]**
   (a) **[10 Points]** What is the tree encoded by the string 77676?

   (b) What is the string encoding the following tree?
   
   A: \[7,7,8,9,9,a,b,b,c\]

3. **Recurrences**  
   **[20 Points]**
   (a) **[10 Points]** Find an asymptotically tight bound for the following recurrence
   
   \[ T(n) = T(n/2) + \log n \]
(b) **[10 Points]** Solve the recurrence \( U(n) = 2U(n/2)U(n/2) \).

4. **Planarity**  
   **[20 Points]**

(a) **[10 Points]** Prove that \( K_5 \) is not a planar graph.

(b) **[10 Points]** Prove that \( K_{3,3} \) is not a planar graph.

5. **Languages**  
   **[20 Points]**

(a) **[10 Points]** Let \( L \) be a regular language, and let \( L^R \) be the reverse language. Namely, \( w = a_1a_2 \ldots a_k \in L^R \), if and only if \( w^R = a_ka_{k-1} \ldots a_1 \in L \). For example, if \( 00011 \in L \), then \( 11000 \in L^R \).  
   Prove that \( L^R \) is a regular language.
(b) [10 Points] Let $L_1, L_2$ be two regular languages. Prove that $L_1 \cap L_2$ is also a regular language (hint: one can prove this directly, or use facts seen in class. In any case, arguing about regular expressions is not a valid solution here, since we did not cover this topic yet.).