Smaller Coresets for k-Median and k-Means Clustering

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Coresets for Clustering

Clustering

P: set of **n** points in $\mathbb{R}^{\mathbf{d}}$, compute a set **C** of **k** centers –

- **k-median**: sum dist. points in **P** to nearest center.
  $\nu_{\mathbf{C}}(\mathbf{P}) = \sum_{\mathbf{p}\in\mathbf{P}} \mathbf{d}(\mathbf{p}, \mathbf{C})$
- **k-means**: sum squares dist. points in **P** to nearest center.

Target: Find **C** that minimizes price.

Coreset

Definition (Coreset)

$\mathcal{S} \subseteq \mathbf{P}$ - A weighted subset is $(\mathbf{k}, \varepsilon)$-coreset if

$$\forall \mathbf{C} \subseteq \mathbb{R}^{\mathbf{d}} \quad \nu_{\mathbf{C}}(\mathbf{P}) \approx_{\varepsilon} \nu_{\mathbf{C}}(\mathcal{S}),$$

where $\mathbf{a} \approx_{\varepsilon} \mathbf{b}$ if $(1-\varepsilon)\mathbf{a} \leq \mathbf{b} \leq (1+\varepsilon)\mathbf{a}$.

Coreset = small sketch of the input for clustering.

Question

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  $O\left(n^{O(1/\varepsilon)}\right)$ time $(1+\varepsilon)$-approximation for points in plane
- **[Kolliopoulos and Rao, 1999]** (Discrete version):
  $O(\varrho n \log n \log k)$ time, where $\varrho = \exp\left[O\left(\varepsilon^{-1}\log 1/\varepsilon\right)^{d-1}\right]$.
- **[Har-Peled and Mazumdar, 2004]**:
  Coreset of size $O(\frac{k \log n}{\varepsilon^d})$
  Improve running time to $O\left(n + \varrho k^{O(1)} \log^{O(1)} n\right)$

New Result:

Show coreset of size $O(k^2/\varepsilon^d)$

Independent of n!

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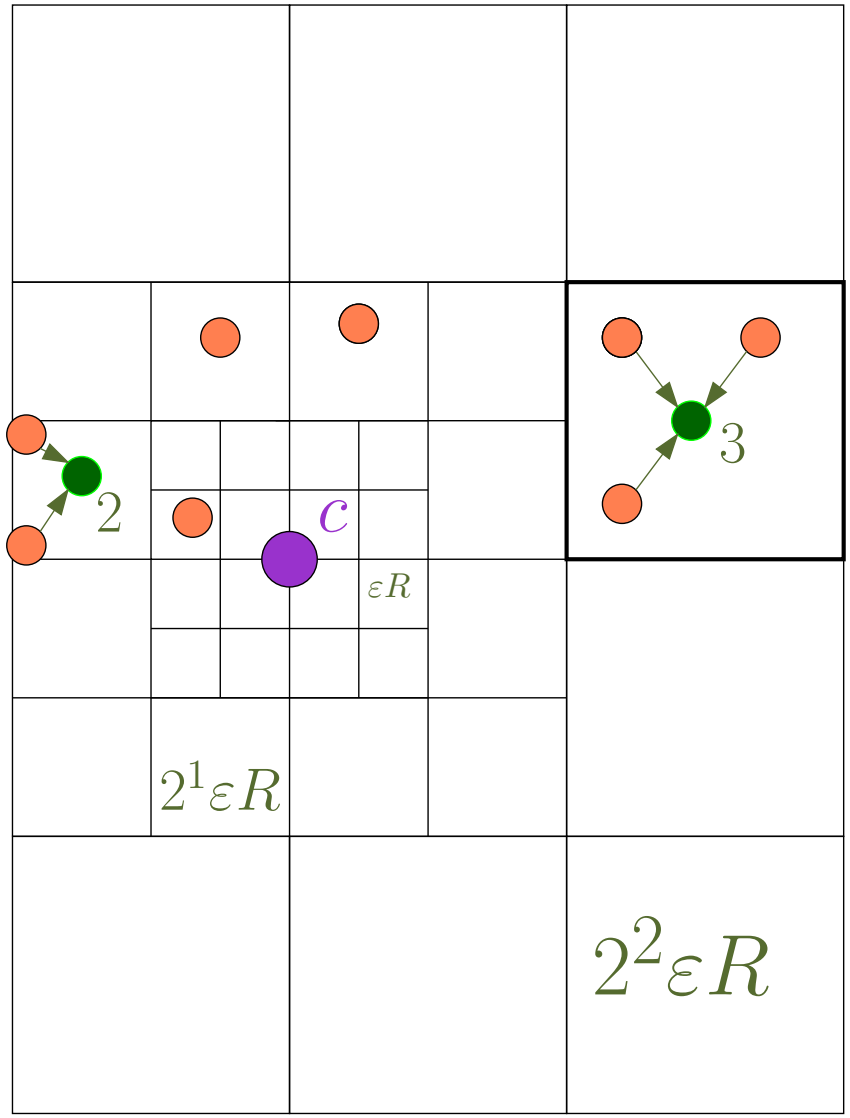
k-means - Effros & Schulman result

- Deterministic clustering with data nets
  [Effros and Schulman, 2003]
- For **k** means only.
- Show the existence of centroid set of size independent of **n**.
- Complicated.
- Inspiration to conjecture that small coresets exists.
- New result implies their result (better parameters).
- New result much simpler.
- Effros & Schulman recently showed existance of small coresets for **k**-means.

Old Coreset Construction

Construction

- **D** - a set of **k** centers $\nu_{\mathbf{D}}(\mathbf{P}) = \mathbf{O}(\nu_{\mathbf{opt}})$.
- build exponential grid of $\mathbf{O(\log n)}$ levels around each center,
- snap the points of **P** to this grid.
- *price of snapping is smaller than* $\varepsilon\nu_{\mathbf{opt}}(\mathbf{P}, \mathbf{k})$



Old Coreset Construction – limitations

Lemma $\Rightarrow$ Problem

If snapping is "cheap" ($\leq \varepsilon \nu_{\mathrm{opt}}(\mathbf{P}, \mathbf{k})$), then $|\mathcal{S}| = \Omega(\log n)$.

Conclusions

For $\mathcal{S}$ to be small

$\Rightarrow$ Must pick $\mathcal{S}$ such that snapping errors *cancel each other out*.

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Warmup exercise - Coreset in One Dimension

Problem

$\mathbf{P} \subseteq \mathbb{R}$ - $\mathbf{n}$ *points on real line.*
Pick $(\mathbf{k}, \varepsilon)$*-coreset for* $\mathbf{P}$ *for centers on the line.*

Basic Idea:

Break the point set into batches, and use the mean point of batch, as the representative for the coreset.

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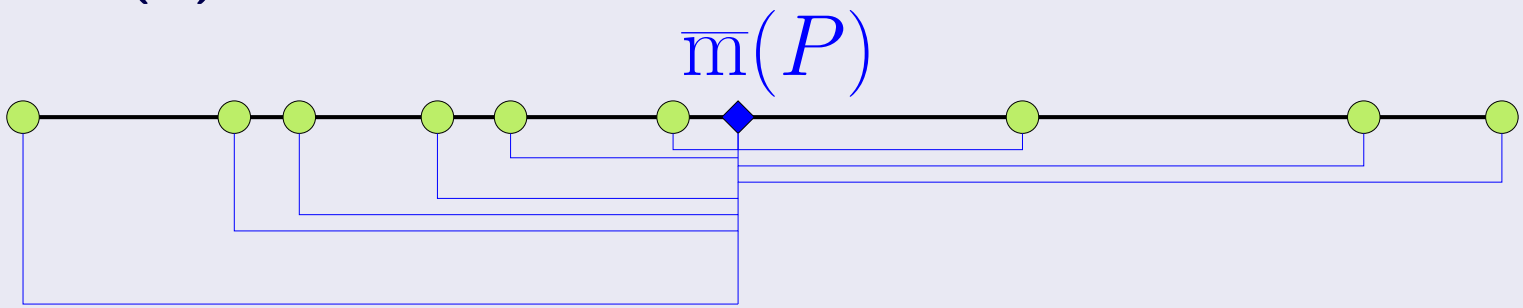
Warmup exercise - Coreset in One Dimension - cont'd

Definition (Cumulative Error)

$\mathcal{E}_\nu(\mathbf{P}) = \nu_{\overline{\mathbf{m}}}(\mathbf{P}) = \sum_{\mathbf{p}\in\mathbf{P}} \mathbf{w_p}\|\mathbf{p}\overline{\mathbf{m}}\|$
$\overline{\mathbf{m}} = \overline{\mathbf{m}}(\mathbf{P})$ is mean of $\mathbf{P}$.



$\mathcal{E}_\nu(\mathbf{P})$ = Total length of blue segments.

Warmup exercise - Coreset in One Dimension - cont'd

- $\mathbf{V}$: $\nu_{\mathrm{opt}}(\mathbf{P}, \mathbf{k}) \leq \mathbf{V} \leq \mathbf{c} \cdot \nu_{\mathrm{opt}}(\mathbf{P}, \mathbf{k})$
- Sweep $\mathbf{P}$ from left to right.
- Break into batches $\mathbf{B_i}$ with $\mathcal{E}_\nu(\mathbf{B_i}) = \frac{\varepsilon}{\mathbf{10ck}}\mathbf{V}$ (allow fractional points)
- # batches is $\alpha = \mathbf{O(k/\varepsilon)}$
- Coreset point: $(\overline{\mathbf{m}}(\mathbf{B_i}), \mathbf{w}(\mathbf{B_i}))$
- Coreset: $\left\{ (\overline{\mathbf{m}}(\mathbf{B_i}), \mathbf{w}(\mathbf{B_i})) \;\middle|\; \mathbf{i} = 1, \ldots, \alpha \right\}$.

Correctness

- $\mathbf{B}$ - a batch of points
- $\mathcal{I}(\mathbf{B})$ - interval containing $\mathbf{B}$.
- $\forall \mathbf{c}$ outside interval $\mathcal{I}(\mathbf{B})$ then $\nu_{\mathbf{c}}(\mathbf{P}) = \nu_{\mathbf{c}}(\mathcal{S})$.
- *problematic* batches = contribute to the error
  - Batches served by two or more centers ($\leq \mathbf{k}-\mathbf{1}$ batches)
  - batches with center point inside them ($\leq \mathbf{k}$ such batches).
- Error of problematic batches $\leq \mathbf{L} = \mathcal{E}_{\nu}(\mathbf{B_i}) = \frac{\varepsilon}{\mathbf{10ck}}\mathbf{V}$
- Total error $\leq (\mathbf{2k}-\mathbf{1}) \cdot \mathbf{L} \leq \varepsilon \nu_{\mathrm{opt}}(\mathbf{P}, \mathbf{k})$.

Construction can be extended to:

- Points on line $\ell \subseteq \mathbb{R}^{\mathbf{d}}$.
- Centers somewhere in $\mathbb{R}^{\mathbf{d}}$
- Extension not trivial (but doable - details in paper)

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What we currently have...

Lemma

- $\mathbf{P}$ *a set of points on a line* ℓ
- ℓ *- line in* $\mathbb{R}^{\mathbf{d}}$
- *Coreset of size* $\mathbf{O(k/\varepsilon)}$ *for* $\mathbf{P}$
- *For* **k***-median clustering.*

Problem

General point-set do not lie on a single line.

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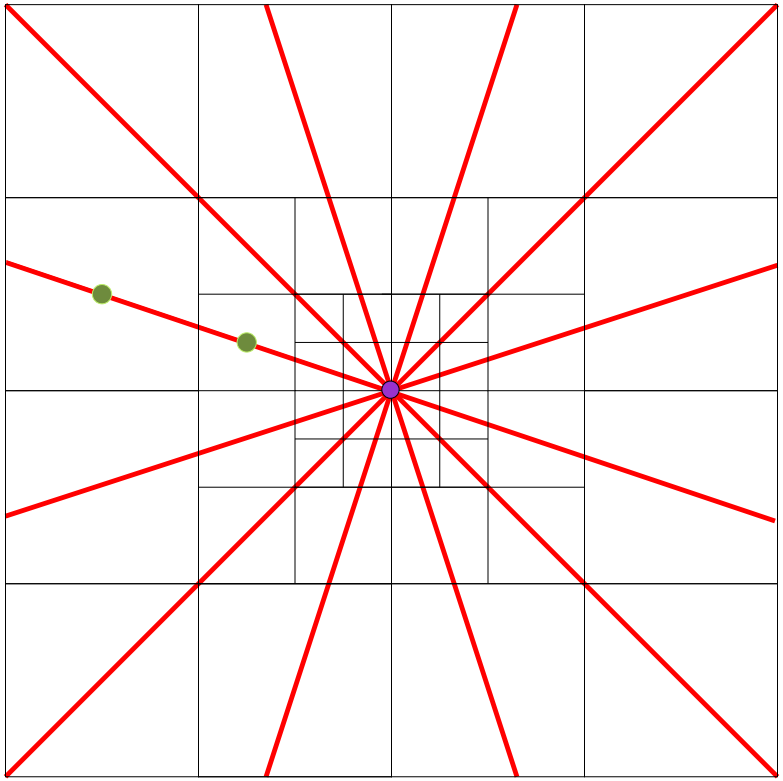

# of grid cell centers: $\mathbf{O(\varepsilon^{-d} \log n)}$.

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Conclusion

We can snap points to a small number of lines.

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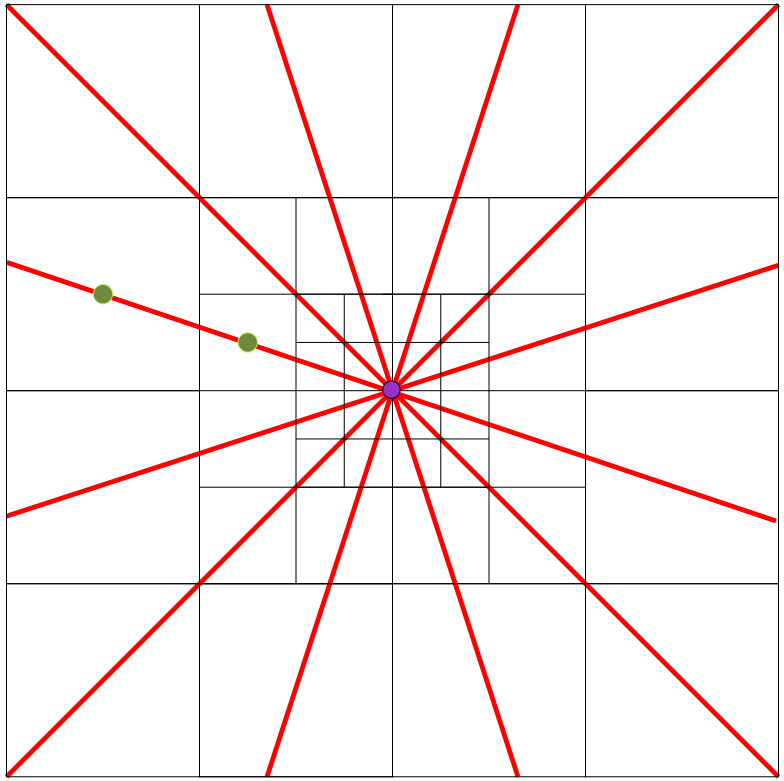

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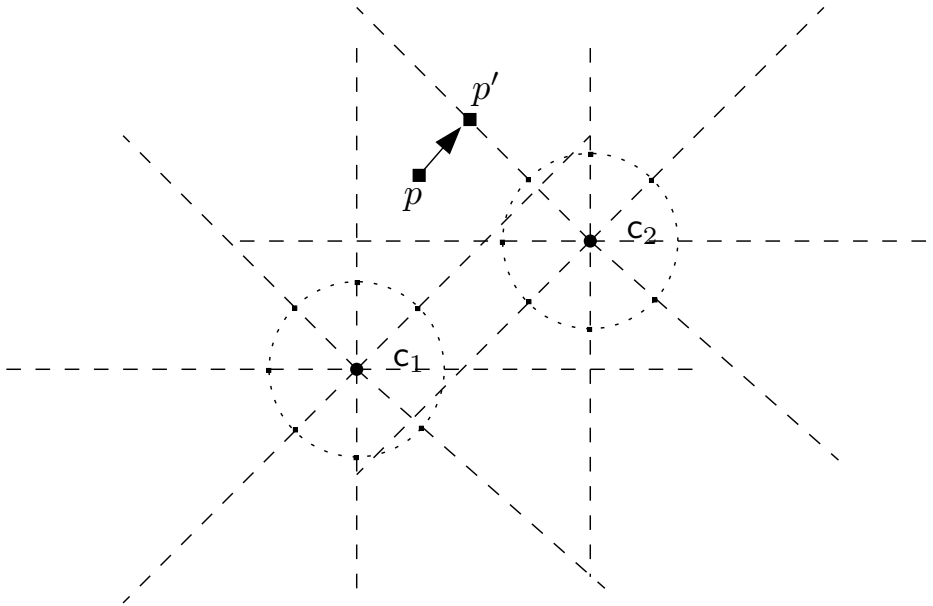
Extending to Higher Dimensions

- comp. $\mathbf{C} = \{\mathbf{c_1}, \ldots, \mathbf{c_k}\}$ s.t. $\nu_{\mathbf{C}}(\mathbf{P}) = \mathbf{O}(\nu_{\mathbf{opt}}(\mathbf{P}, \mathbf{k}))$
- construct a fan of lines around points of $\mathbf{C}$
- snap points of $\mathbf{P}$ to the closest line
- build coreset for each set on a line.

Correctness



- Number of lines is $\mathbf{O(\frac{k}{\varepsilon^{d-1}})} \Rightarrow$ coreset size is $\mathbf{O(\frac{k^2}{\varepsilon^d})}$
- snapping error is bounded by $\frac{\varepsilon}{3}\nu_{\mathbf{opt}}(\mathbf{P}, \mathbf{k})$
- error by construction $\frac{\varepsilon}{3}\nu_{\mathbf{opt}}(\mathbf{P'}, \mathbf{k})$ ($\mathbf{P'}$ are snapped points)
- Total error $\leq \varepsilon\nu_{\mathbf{opt}}(\mathbf{P}, \mathbf{k})$

k-means: coreset construction

- Similar scheme works for **k**-means.
- Analysis slightly simpler.
- $\Rightarrow$ Coreset size $\mathbf{O(k^3/\varepsilon^{d+1})}$
- Full details in the paper.

k-means: results

Definition (Centroid Set)

$\mathcal{D} \subseteq \mathbb{R}^{\mathbf{d}}$: $(\mathbf{k}, \varepsilon)$-*approximate centroid set* for $\mathbf{P}$, if $\exists \mathbf{C} \subseteq \mathcal{D}$ such that $\mu_{\mathbf{C}}(\mathbf{P}) \leq (1+\varepsilon)\mu_{\mathbf{opt}}(\mathbf{P}, \mathbf{k})$

- Matoušek **[Matoušek, 2000]**: $\exists$ an ε-approx. centroid set. Size $\mathbf{O}(\mathbf{n}\varepsilon^{-\mathbf{d}} \log(1/\varepsilon))$
- **[Effros and Schulman, 2003]** Centroid set of size $\mathbf{O}\left(\varepsilon^{-\mathbf{d}-\mathbf{1}}(\mathbf{k}^{\mathbf{4}} + \mathbf{k}^{\mathbf{2}}\varepsilon^{-\mathbf{2}})\right)$.

New Result:

A $(\mathbf{k}, \varepsilon)$-centroid set for $\mathbf{k}$-means clustering with size $\mathbf{O}\left((\mathbf{k}^{\mathbf{3}}/\varepsilon^{\mathbf{2d+1}}) \log(1/\varepsilon)\right)$.
Fast running time...

Minor improvement in RT for $\mathbf{k}$-means clustering (over **[Har-Peled and Mazumdar, 2004]**)

Open Problems

- Improve running time of approx. **k**-means clustering?
- FPTAS for **k**-median and **k**-means (in both **k** and $\mathbf{1}/\varepsilon$)?
- A coreset with only polynomial dependency on the dimension and no dependency on **n**?
  Relevant results **[Bădoiu et al., 2002, Chen, 2004]**.
- Improve dependency on **k** and ε in coresets size?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