Quasi-Polynomial Time Approximation Scheme for Sparse Subsets of Polygons

Sariel Har-Peled[1]

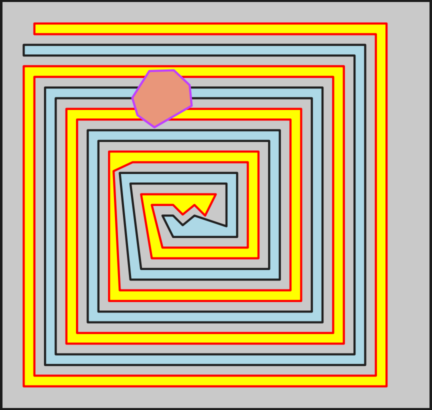
[1]UIUC, Illinois, USA, Earth, The Milky Way, This Universe (Hopefully)

Part I

The problem & main result

2: The problem

- $\mathcal{P}$: m polygons in the plane.
- n: Total complexity of $\mathcal{P}$.



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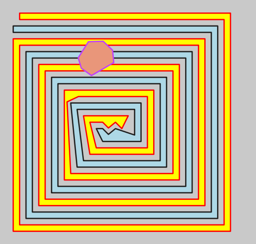

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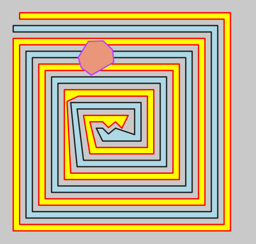
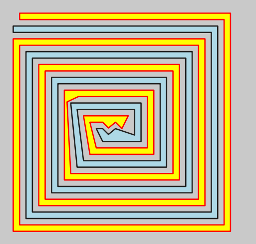
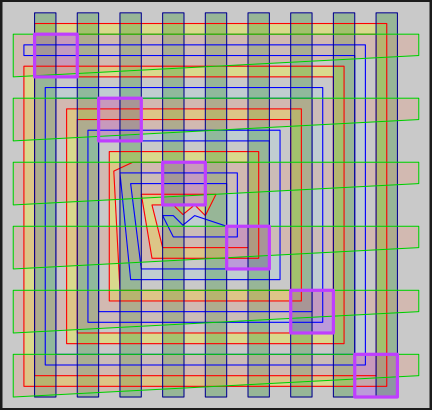




3: The problem - another example



4: Main result

Theorem

1. $\mathcal{P}$: set m simple weighted polygons.
2. n: total complexity.
3. Compute an independent set weight $\geq (1-\varepsilon)W_{\text{opt}}$.
4. W_{opt}: maximum weight of optimal independent set.
5. Running time: $O\left(m^{\text{poly}(\log m, 1/\varepsilon)} + m^{O(1)}n\right)$.

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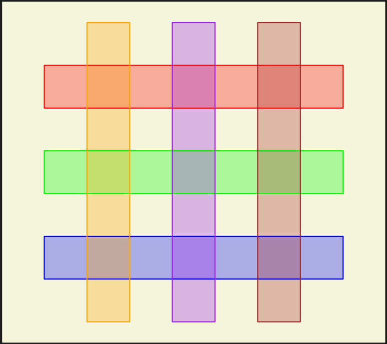
Contribution: Polygons of arbitrary complexity!

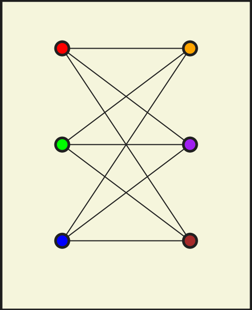
Part II

Background and previous work

5: Intersection graph

Find independent set in the graph.





6: Computing independent set

What is known...

1. **NP-Complete**.
2. No $|V|^{1-\varepsilon}$-approx if $\mathbf{NP} \neq \mathbf{ZPP}$ **[Hastad, 1996]**.
3. Maximum degree of graph is $\leq \mathbf{3}$: no **PTAS** possible.

7: Geometric settings

What is known...

1. **PTAS**es for fat objects (disks/squares).
   1. [Chan, 2003, Erlebach, Jansen, and Seidel, 2005]: Quadtrees + random shifting
   2. [Chan, 2003]: Uses planar separator (unweighted).
2. Axis-parallel rectangles.
   1. [Chalermsook and Chuzhoy, 2009]: $O(\log \log n)$ approximation.
   2. [Chan and Har-Peled, 2012]: $O(\log n / \log \log n)$-approx for weighted case.
3. Pseudo-disks
   [Chan and Har-Peled, 2012]:
   $(1 - \varepsilon)$-approximation (**PTAS**).
   Local search + separator theorem.

8: Geometric settings

Continued...

- Line segments
  1. [Agarwal and Mustafa, 2006]: $O(\sqrt{n_{opt}})$-approx. Dilworth's theorem.
  2. [Fox and Pach, 2011]: n^{ε}-approx. Idea: Large biclique if dense, and cheap separator if sparse.
- New results: [Adamaszek and Wiese, 2013, 2014].
  1. Good news: $(1-\varepsilon)$-approximation for rectangles.
  2. Bad news: Running time $n^{(\log n/\varepsilon)^{O(1)}}$. **QPTAS**: Quasi-polynomial time approximation scheme
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9: Thinking about these complexity classes

Polynomial time	$\exp(O(\log n))$
FPTAS	$\exp\left(O\left(\log \frac{n}{\varepsilon}\right)\right)$
PTAS	$\exp\left(O\left(f(\varepsilon) \log n\right)\right)$
QPTAS	$\exp\left(f(\varepsilon) \log^{O(1)} n\right)$
ETH Exponential Time Hypothesis	$\text{3SAT} \in \exp\left(n^{\Omega(1)}\right)$
EXP	$\exp\left(n^{O(1)}\right)$

Part III

Main Result II

QPTAS for sparse thingies

10: Main result II

Theorem

- $\mathcal{P}$: weighted set of m polygons.
- n: total complexity
- every pair of polygons intersects $O(1)$ times.
- $\Pi_{\mathcal{P}}$: hereditary + sparse + mergeable property
  + exponential time checkable.
- W_{opt}: maximum weight of a set in $\Pi_{\mathcal{P}}$.
- $\implies$ **QPTAS** to compute a subset $X \subseteq \mathcal{P}$
  $X \in \Pi_{\mathcal{P}}$, and $\omega(X) \geq (1-\varepsilon)W_{\mathrm{opt}}$,

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11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $G_{\mathcal{O}}$ is planar.
3. $G_{\mathcal{O}}$ has low genus.
4. $G_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

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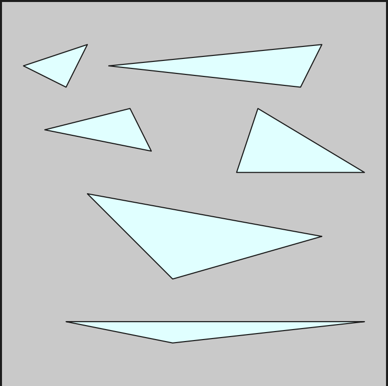
Part IV

Simple decompositions for polygons

Deep in the technicalities zone

12: Vertical decomposition for triangles

For disjoint triangles



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3. Every trapezoid defined by 4 triangles.
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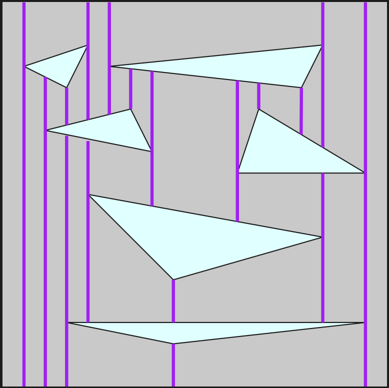

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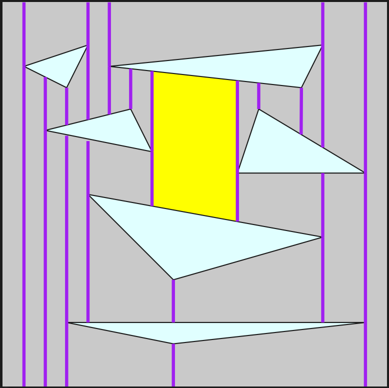

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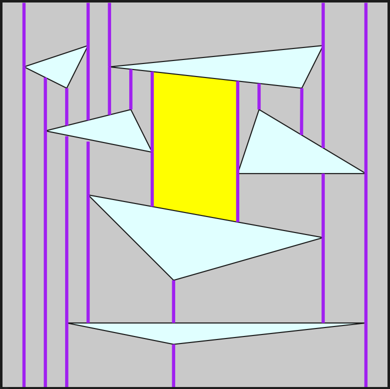

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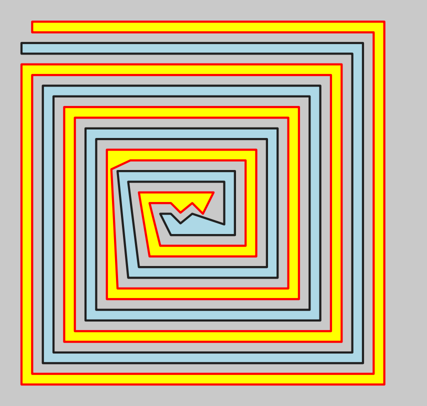

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- Want:
  - Decomposition into $O(m)$ cells.
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- No hope with vertical decomposition.
- Need a more topological approach.

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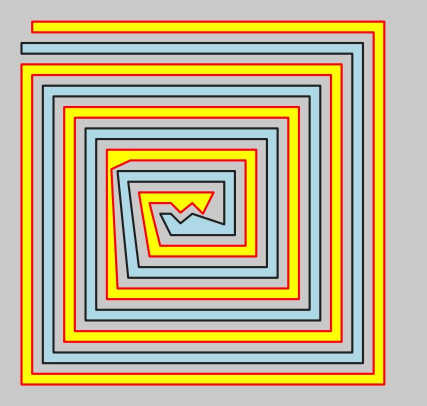

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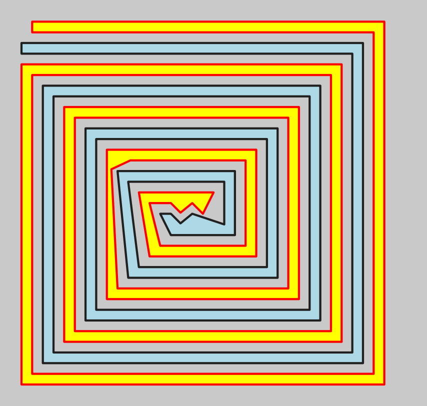

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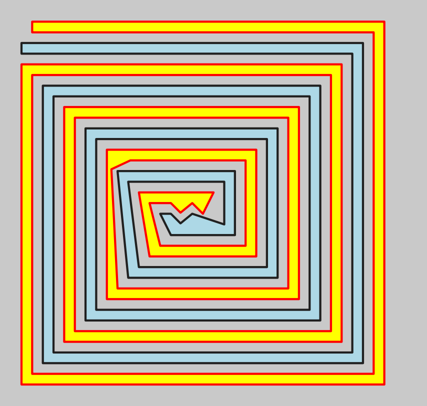

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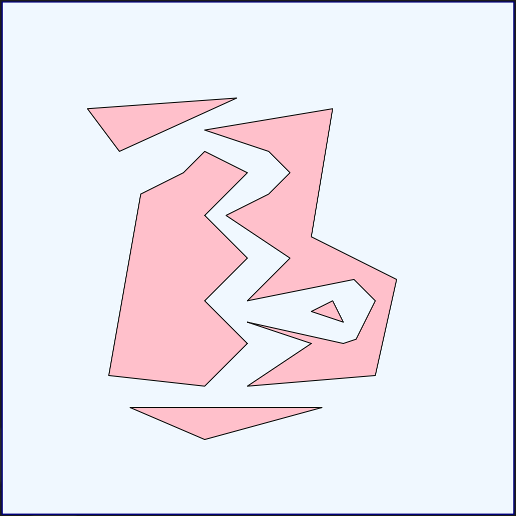
14: Corridor decomposition

Breaking into cells for polygons



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- Critical squares.
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- ...add spokes.
- ...broke comp. into **corridors**.
  - Def. by 4 polygons.
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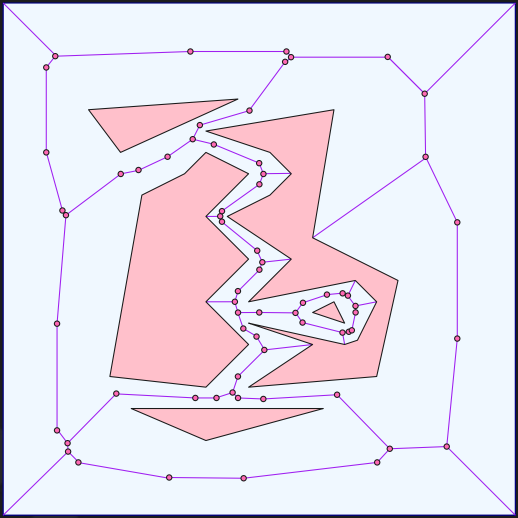

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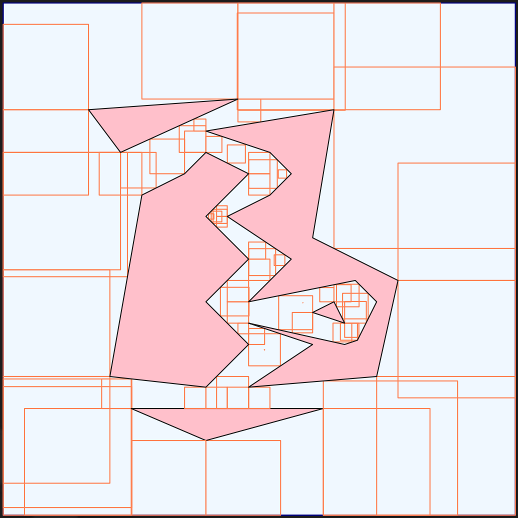

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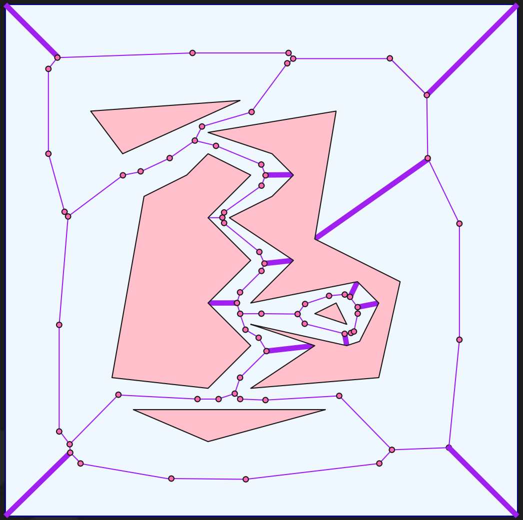

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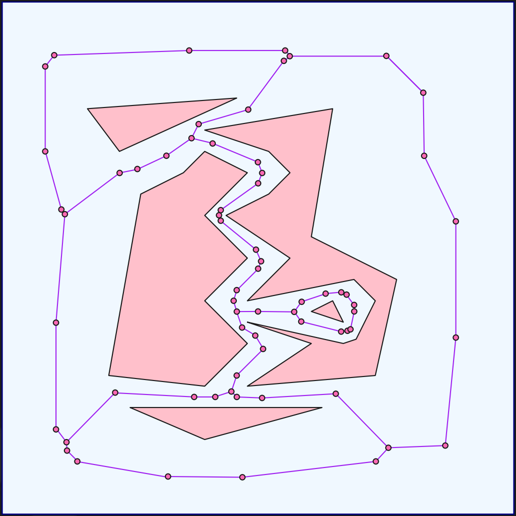

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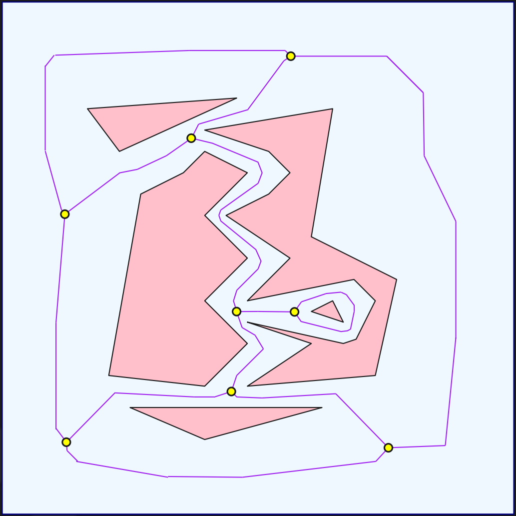

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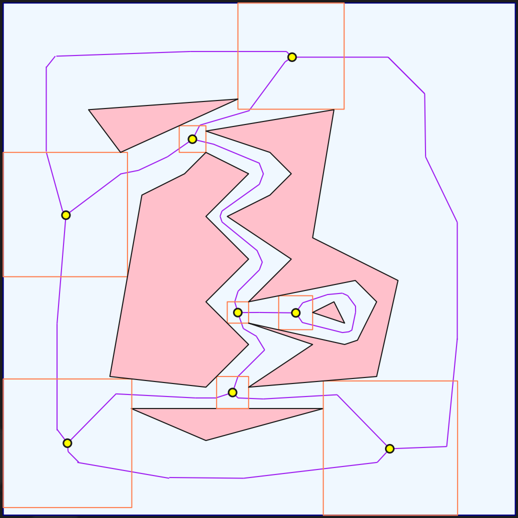

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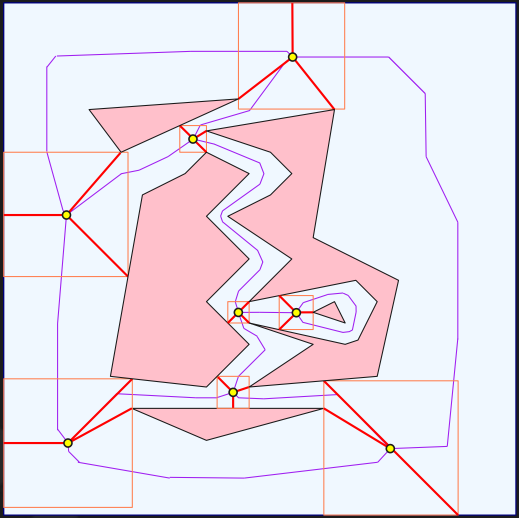

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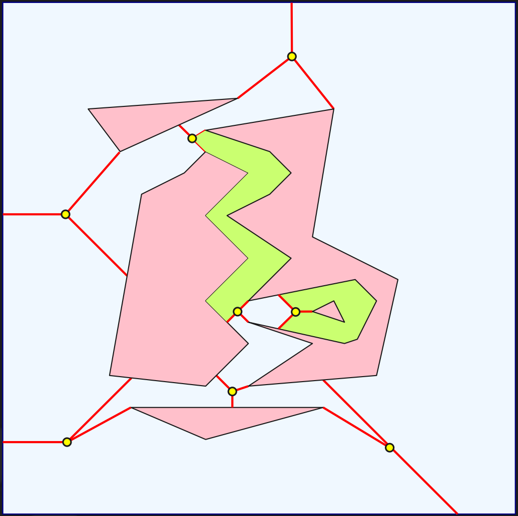

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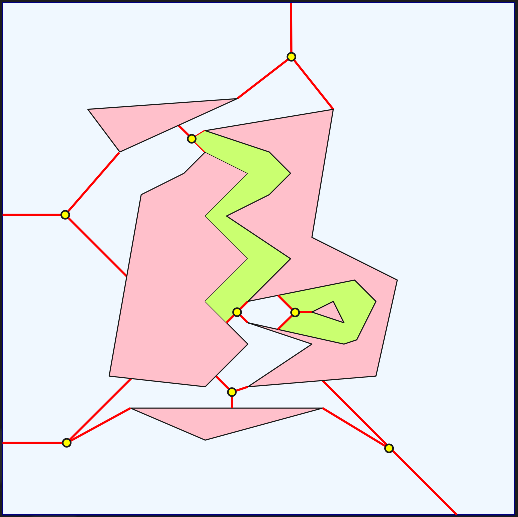

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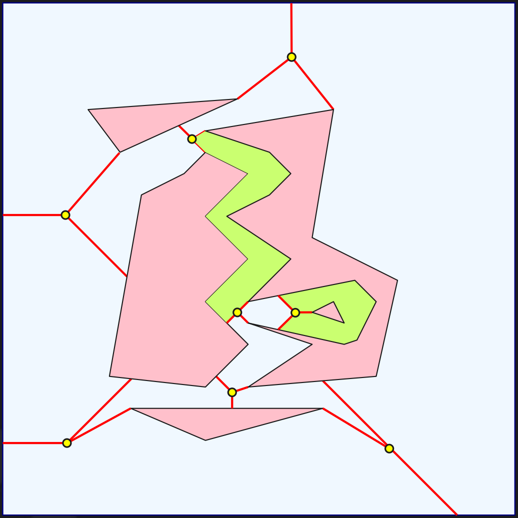

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Part V

Cuttings

Deep in the technicalities zone

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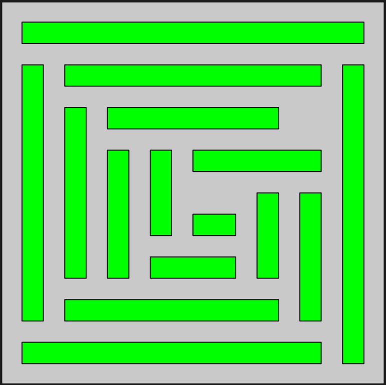
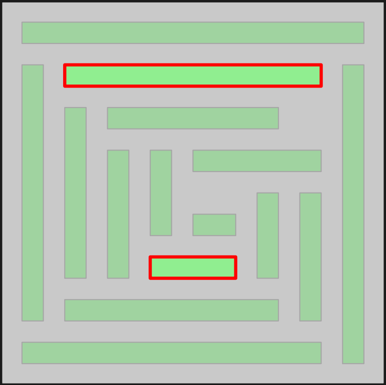


- $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
  r: Parameter
- **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
- **[Chazelle and Friedman, 1990]**
  $1/r$-cutting with $O(r)$ cells.

15: Cuttings



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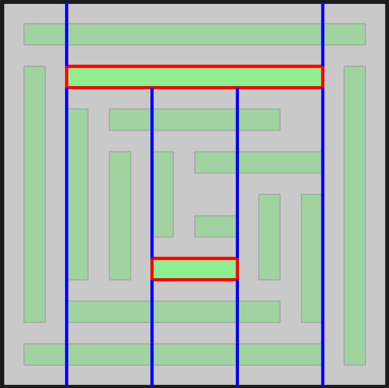

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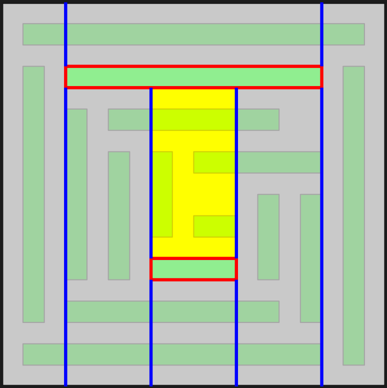

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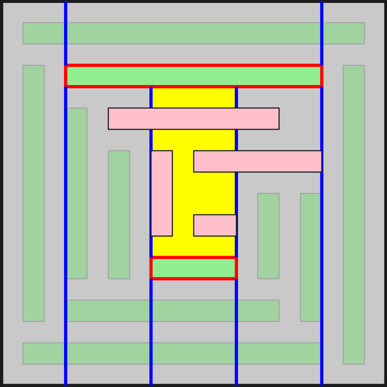

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$\mathcal{P}$: *disjoint weighted polygons*
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Part VI

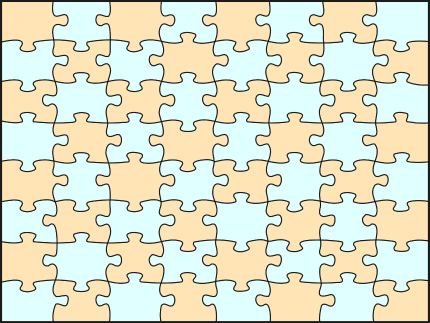
The Approximation Algorithm

A monstrous dynamic programming in action

17: Recursing on optimal solution

- $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
- $1/r$-cutting, with $\widetilde{O}(r)$ cells.
- Tiles form a planar graph.
- tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
- Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
- Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
- Recurse: In : Out

17: Recursing on optimal solution



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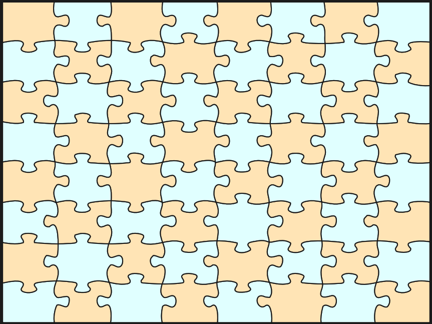

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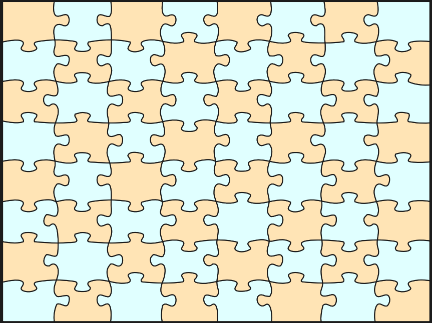

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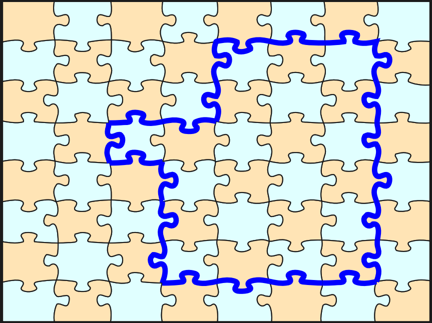

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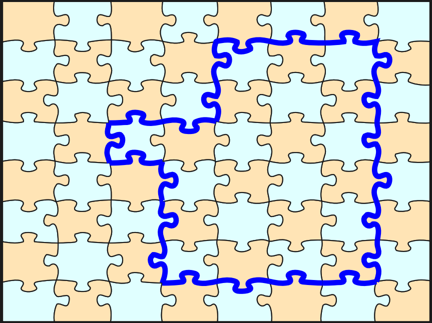

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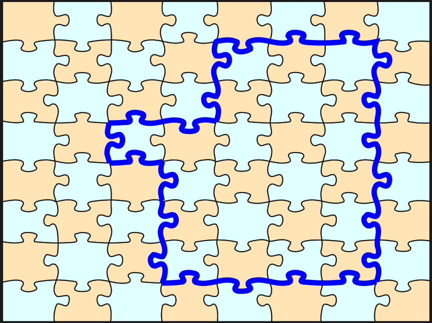

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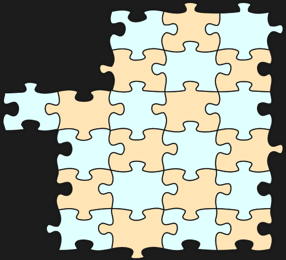

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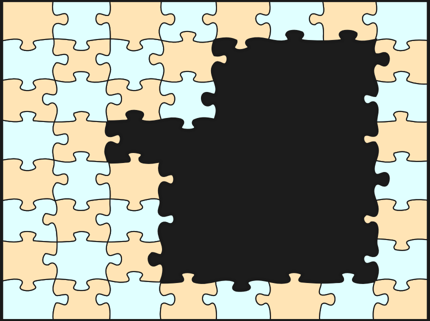

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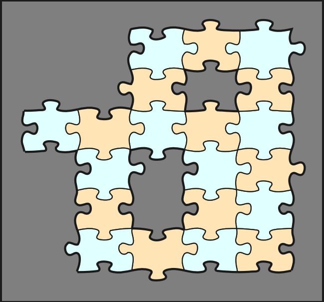
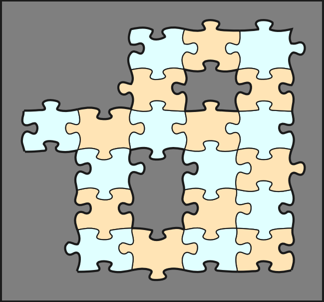
18: How a subproblem looks like



1. $\mathcal{O}$: Opt sol $|\mathcal{O}| \leq m$.
2. Subproblem defined by $O(\log m)$ cycles.
3. Each cycle has $\widetilde{O}(\sqrt{r})$ "corridor" edges.

19: Guessing the separator



- $\mathcal{O}$: Optimal solution
  r: Param (specified shortly).
- $m = |\mathcal{P}|$.
- $m^{O(1)}$ possible edges. (cuttings+corridors decomposition)
- Separator has $\widetilde{O}(\sqrt{r})$ edges.
- $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
- Depth of recursion $O(\log m)$.
- $m^{\widetilde{O}(\sqrt{r} \log m)}$ different subproblems overall.

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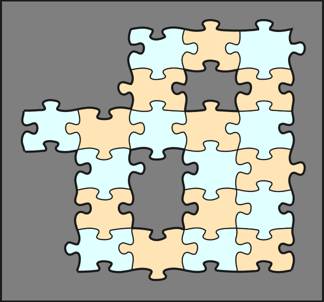

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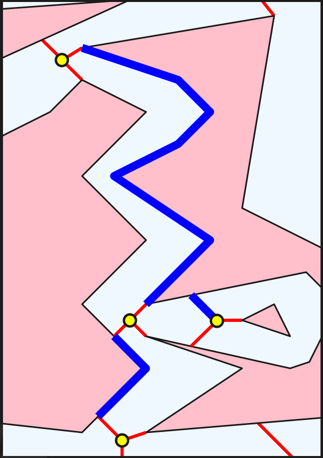

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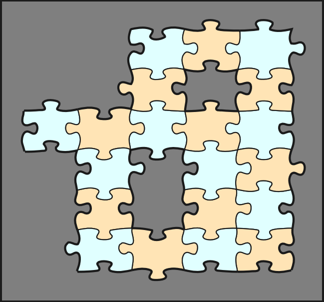

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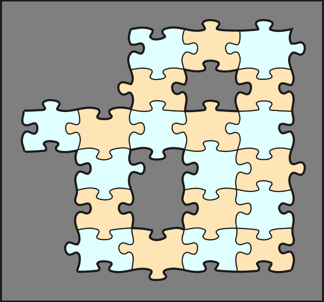

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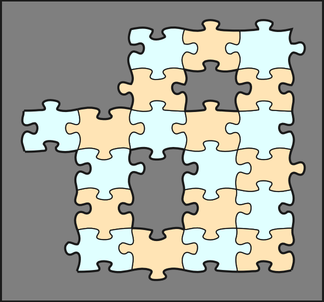

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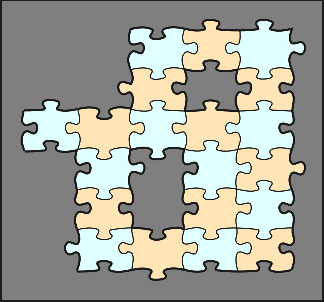

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1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1-\varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r}\log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

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Part VII

Conclusions

This too shell pass

21: Conclusions

1. Framework of [Adamaszek and Wiese, 2013, 2014].
   Insight: Using planar separator on cuttings.
2. In this paper:
   1. Extended framework...
   2. Showed **QPTAS** for independent set of polygons of arbitrary complexity.
   3. Showed techniques works for computing sparse subsets.
3. Open questions for future research:
   1. **PTAS**?
   2. Better running time.


FINIS

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