Quasi-Polynomial Time Approximation Scheme for Sparse Subsets of Polygons

Sariel Har-Peled[1]

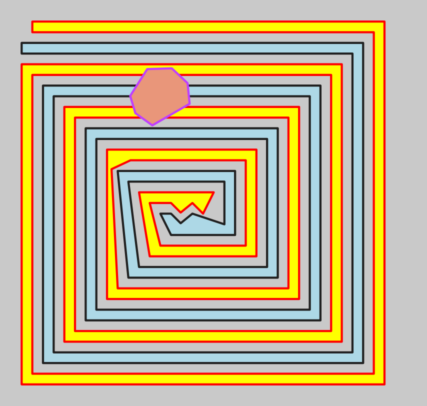
[1]UIUC, Illinois, USA, Earth, The Milky Way, This Universe (Hopefully)

Part I

The problem & main result

2: The problem

- $\mathcal{P}$: m polygons in the plane.
- n: Total complexity of $\mathcal{P}$.

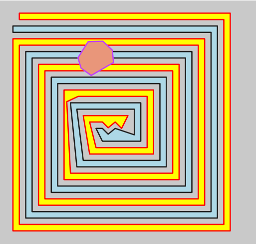


2: The problem

- $\mathcal{P}$: m polygons in the plane.
- n: Total complexity of $\mathcal{P}$.

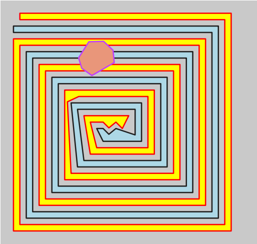
Task...

Compute largest independent set of polygons in $\mathcal{P}$.



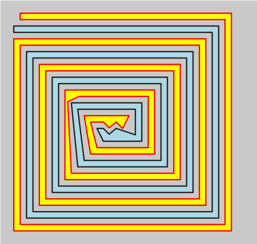
2: The problem

- $\mathcal{P}$: m polygons in the plane.
- n: Total complexity of $\mathcal{P}$.

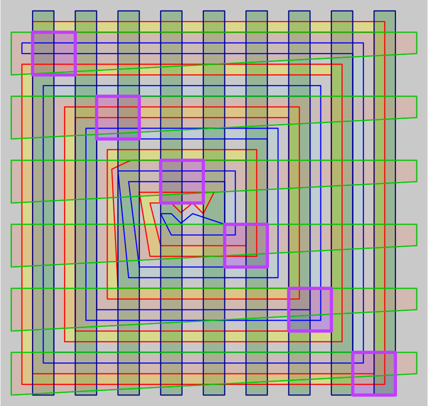


Task...

Compute largest independent set of polygons in $\mathcal{P}$.



3: The problem - another example



4: Main result

Theorem

1. $\mathcal{P}$: set m simple weighted polygons.
2. n: total complexity.
3. Compute an independent set weight $\geq (1 - \varepsilon) W_{\text{opt}}$.
4. W_{opt}: maximum weight of optimal independent set.
5. Running time: $O\left(m^{\text{poly}(\log m, 1/\varepsilon)} + m^{O(1)} n\right)$.

4: Main result

Theorem

1. $\mathcal{P}$: set m simple weighted polygons.
2. n: total complexity.
3. Compute an independent set weight $\geq (1 - \varepsilon)W_{\mathrm{opt}}$.
4. W_{opt}: maximum weight of optimal independent set.
5. Running time: $O\left(m^{\mathrm{poly}(\log m, 1/\varepsilon)} + m^{O(1)} n\right)$.

4: Main result

Theorem

1. $\mathcal{P}$: set m simple weighted polygons.
2. n: total complexity.
3. Compute an independent set weight $\geq (1-\varepsilon)W_{\text{opt}}$.
4. W_{opt}: maximum weight of optimal independent set.
5. Running time: $O\left(m^{\text{poly}(\log m, 1/\varepsilon)} + m^{O(1)} n\right)$.

4: Main result

Theorem

1. $\mathcal{P}$: set m simple weighted polygons.
2. n: total complexity.
3. Compute an independent set weight $\geq (1-\varepsilon)W_{\mathrm{opt}}$.
4. W_{opt}: maximum weight of optimal independent set.
5. Running time: $O\left(m^{\mathrm{poly}(\log m, 1/\varepsilon)} + m^{O(1)} n\right)$.

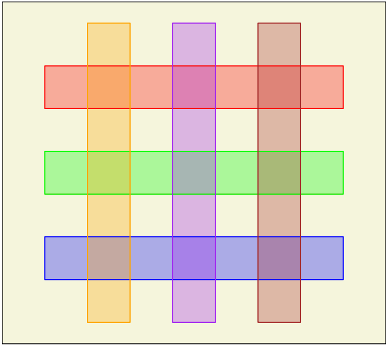
Contribution: Polygons of arbitrary complexity!

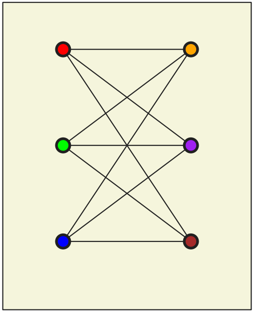
Part II

Background and previous work

5: Intersection graph

Find independent set in the graph.





1. **NP-Complete**.
2. No $|V|^{1-\varepsilon}$-approx if **NP** $\neq$ **ZPP** **[Hastad, 1996]**.
3. Maximum degree of graph is $\leq \mathbf{3}$: no **PTAS** possible.

7: Geometric settings

What is known...

1. **PTAS**es for fat objects (disks/squares).
    1. **[Chan, 2003, Erlebach, Jansen, and Seidel, 2005]**: Quadtrees + random shifting
    2. **[Chan, 2003]**: Uses planar separator (unweighted).
2. Axis-parallel rectangles.
    1. **[Chalermsook and Chuzhoy, 2009]**: $O(\log \log n)$ approximation.
    2. **[Chan and Har-Peled, 2012]**: $O(\log n / \log \log n)$-approx for weighted case.
3. Pseudo-disks
   [Chan and Har-Peled, 2012]:
   $(1 - \varepsilon)$-approximation (**PTAS**).
   Local search + separator theorem.

8: Geometric settings
Continued...

1. Line segments
   1. **[Agarwal and Mustafa, 2006]**: $O\left(\sqrt{n_{\mathrm{opt}}}\right)$-approx.
      Dilworth's theorem.
   2. **[Fox and Pach, 2011]**: n^{ε}-approx.
      Idea: Large biclique if dense, and cheap separator if sparse.
2. New results: **[Adamaszek and Wiese, 2013, 2014]**.
   1. Good news: $(1-\varepsilon)$-approximation for rectangles.
   2. Bad news: Running time $n^{(\log n/\varepsilon)^{O(1)}}$.
      QPTAS: Quasi-polynomial time approximation scheme
   3. Main idea: Cuttings + planar separator.

1. Line segments
   1. [Agarwal and Mustafa, 2006]: $O\left(\sqrt{n_{\mathrm{opt}}}\right)$-approx.
      Dilworth's theorem.
   2. [Fox and Pach, 2011]: n^{ε}-approx.
      Idea: Large biclique if dense, and cheap separator if sparse.
2. New results: [Adamaszek and Wiese, 2013, 2014].
   1. Good news: $(1-\varepsilon)$-approximation for rectangles.
   2. Bad news: Running time $n^{(\log n/\varepsilon)^{O(1)}}$.
      QPTAS: Quasi-polynomial time approximation scheme
   3. Main idea: Cuttings + planar separator.

9: Thinking about these complexity classes

Polynomial time	$\exp(O(\log n))$
FPTAS	$\exp\left(O\left(\log \frac{n}{\varepsilon}\right)\right)$
PTAS	$\exp\Big(O\Big(f(\varepsilon)\log n\Big)\Big)$
QPTAS	$\exp\Big(f(\varepsilon)\log^{O(1)} n\Big)$
ETH Exponential Time Hypothesis	3SAT $\in \exp\Big(n^{\Omega(1)}\Big)$
EXP	$\exp\Big(n^{O(1)}\Big)$

Part III

Main Result II

QPTAS for sparse thingies

10: Main result II

Theorem

1. $\mathcal{P}$: weighted set of m polygons.
2. n: total complexity
3. every pair of polygons intersects $O(1)$ times.
4. $\Pi_{\mathcal{P}}$: hereditary + sparse + mergeable property + exponential time checkable.
5. W_{opt}: maximum weight of a set in $\Pi_{\mathcal{P}}$.
6. $\implies$ **QPTAS** to compute a subset $X \subseteq \mathcal{P}$ $X \in \Pi_{\mathcal{P}}$, and $\omega(X) \geq (1 - \varepsilon)W_{\mathrm{opt}}$,

10: Main result II

Theorem

1. $\mathcal{P}$: *weighted set of* m *polygons.*
2. n: *total complexity*
3. *every pair of polygons intersects* $O(1)$ *times.*
4. $\Pi_{\mathcal{P}}$: *hereditary* $+$ *sparse* $+$ *mergeable property* $+$ *exponential time checkable.*
5. W_{opt}: *maximum weight of a set in* $\Pi_{\mathcal{P}}$.
6. $\implies$ **QPTAS** *to compute a subset* $X \subseteq \mathcal{P}$ $X \in \Pi_{\mathcal{P}}$, *and* $\omega(X) \geq (1-\varepsilon)W_{\mathrm{opt}}$,

10: Main result II

Theorem

1. $\mathcal{P}$: *weighted set of* m *polygons.*
2. n: *total complexity*
3. *every pair of polygons intersects* $O(1)$ *times.*
4. $\Pi_{\mathcal{P}}$: *hereditary* $+$ *sparse* $+$ *mergeable property* $+$ *exponential time checkable.*
5. W_{opt}: *maximum weight of a set in* $\Pi_{\mathcal{P}}$.
6. $\implies$ **QPTAS** *to compute a subset* $X \subseteq \mathcal{P}$ $X \in \Pi_{\mathcal{P}}$, *and* $\omega(X) \geq (1 - \varepsilon) W_{\mathrm{opt}}$,

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.
QPTAS for $(1 - \varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $G_{\mathcal{O}}$ is planar.
3. $G_{\mathcal{O}}$ has low genus.
4. $G_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$, c a constant.
   $c = 1$: Independent set.
2. $G_{\mathcal{O}}$ is planar.
3. $G_{\mathcal{O}}$ has low genus.
4. $G_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $\mathbf{G}_{\mathcal{O}}$ is planar.
3. $\mathbf{G}_{\mathcal{O}}$ has low genus.
4. $\mathbf{G}_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$, c a constant.
   $c = 1$: Independent set.
2. $\mathbf{G}_{\mathcal{O}}$ is planar.
3. $\mathbf{G}_{\mathcal{O}}$ has low genus.
4. $\mathbf{G}_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.
QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $\mathbf{G}_{\mathcal{O}}$ is planar.
3. $\mathbf{G}_{\mathcal{O}}$ has low genus.
4. $\mathbf{G}_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $\mathbf{G}_{\mathcal{O}}$ is planar.
3. $\mathbf{G}_{\mathcal{O}}$ has low genus.
4. $\mathbf{G}_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

11: Implications of Main Result II

$\mathcal{P}$: Set of polygons/pseudo-disks.

QPTAS for $(1-\varepsilon)$-approx to heaviest subset $\mathcal{O}$ of $\mathcal{P}$ s.t.

1. Pseudo-disks: every point covered $\leq c$ times by $\mathcal{O}$,
   c a constant.
   $c = 1$: Independent set.
2. $\mathbf{G}_{\mathcal{O}}$ is planar.
3. $\mathbf{G}_{\mathcal{O}}$ has low genus.
4. $\mathbf{G}_{\mathcal{O}}$ does not contain $K_{s,t}$ as a subgraph. s and t constants.

Main point...

Independent set is just one possible sparse subset one can compute with these techniques.

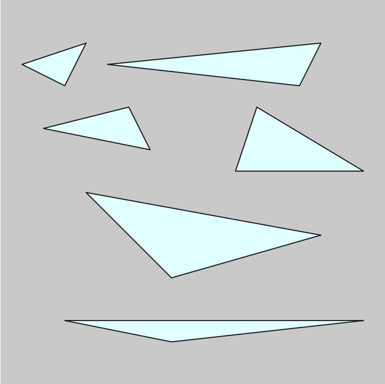
Part IV

Simple decompositions for polygons

Deep in the technicalities zone

12: Vertical decomposition for triangles

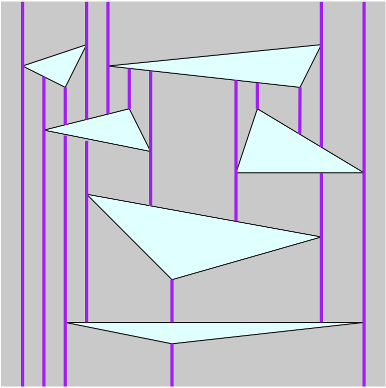
For disjoint triangles



1. m disjoint triangles.
2. Rays up/down vertices. Get $O(m)$ v-traps.
3. Every trapezoid defined by 4 triangles.
4. **Q:** How to do VD for polygons?

12: Vertical decomposition for triangles

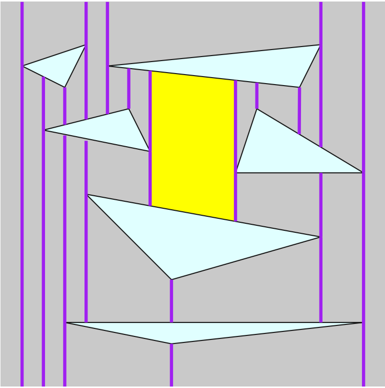
For disjoint triangles



1. m disjoint triangles.
2. Rays up/down vertices. Get $O(m)$ v-traps.
3. Every trapezoid defined by 4 triangles.
4. **Q:** How to do VD for polygons?

12: Vertical decomposition for triangles

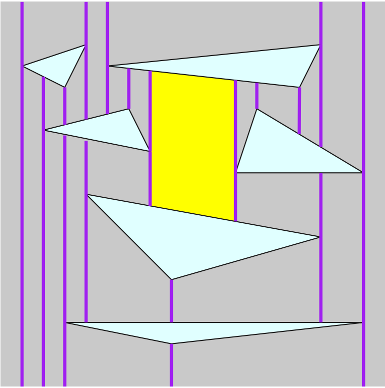
For disjoint triangles



1. m disjoint triangles.
2. Rays up/down vertices.
   Get $O(m)$ v-traps.
3. Every trapezoid defined by 4 triangles.
4. **Q:** How to do VD for polygons?

12: Vertical decomposition for triangles

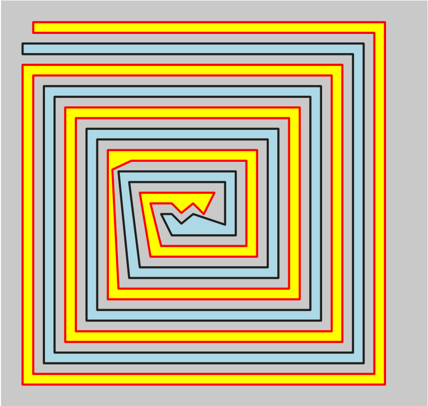
For disjoint triangles



1. m disjoint triangles.
2. Rays up/down vertices.
   Get $O(m)$ v-traps.
3. Every trapezoid defined by **4** triangles.
4. **Q:** How to do VD for polygons?

13: Vertical decomposition for polygons

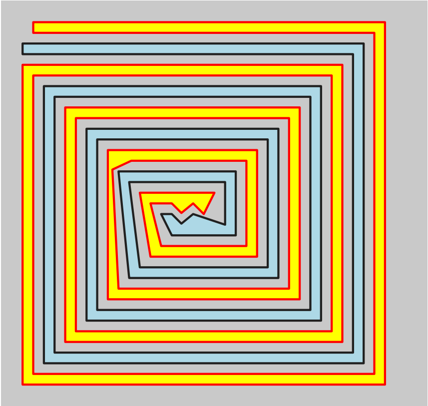
For disjoint polygons



1. m disjoint polygons. n: total complexity of polygons.
2. Want:
   1. Decomposition into $O(m)$ cells.
   2. Cell def. by $O(1)$ polygons.
3. No hope with vertical decomposition.
4. Need a more topological approach.

13: Vertical decomposition for polygons

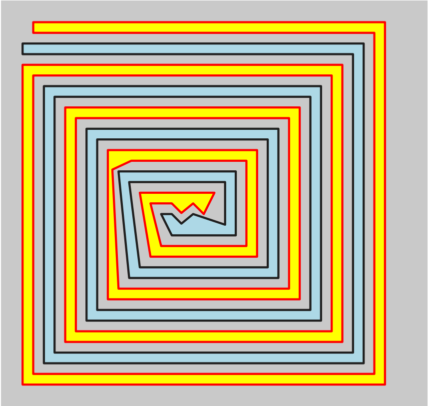
For disjoint polygons



1. m disjoint polygons. n: total complexity of polygons.
2. Want:
   1. Decomposition into $O(m)$ cells.
   2. Cell def. by $O(1)$ polygons.
3. No hope with vertical decomposition.
4. Need a more topological approach.

13: Vertical decomposition for polygons

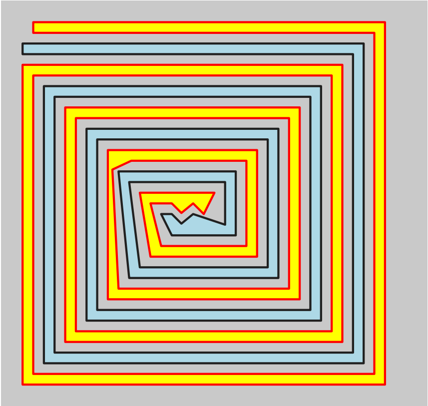
For disjoint polygons



1. m disjoint polygons. n: total complexity of polygons.
2. Want:
   1. Decomposition into $O(m)$ cells.
   2. Cell def. by $O(1)$ polygons.
3. No hope with vertical decomposition.
4. Need a more topological approach.

13: Vertical decomposition for polygons

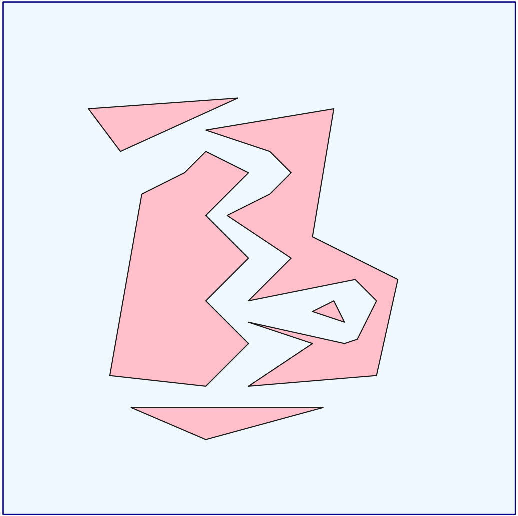
For disjoint polygons



1. m disjoint polygons. n: total complexity of polygons.
2. Want:
    1. Decomposition into $O(m)$ cells.
    2. Cell def. by $O(1)$ polygons.
3. No hope with vertical decomposition.
4. Need a more topological approach.

14: Corridor decomposition

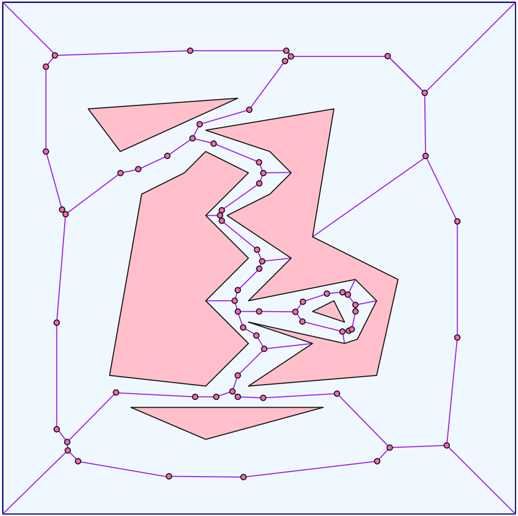
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

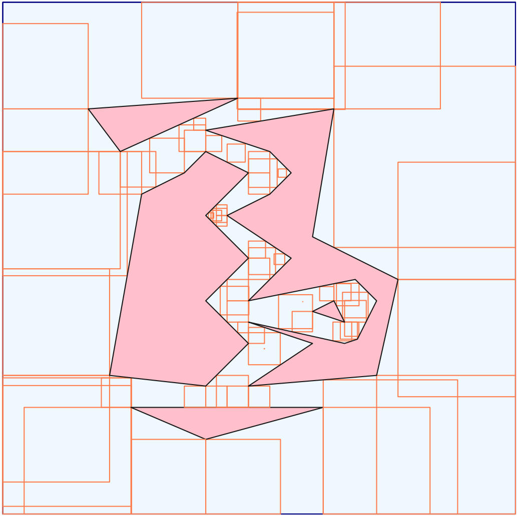
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

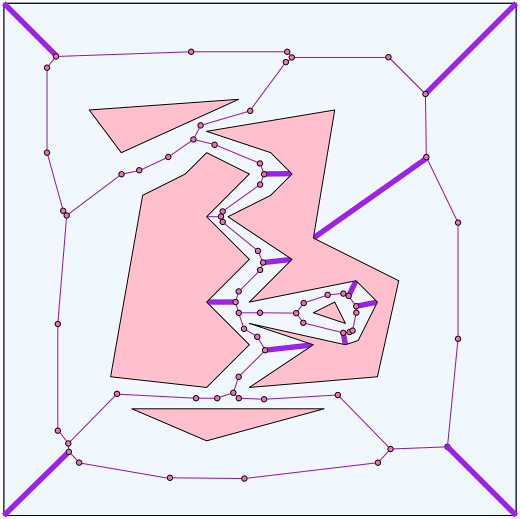
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
   1. Def. by 4 polygons.
   2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

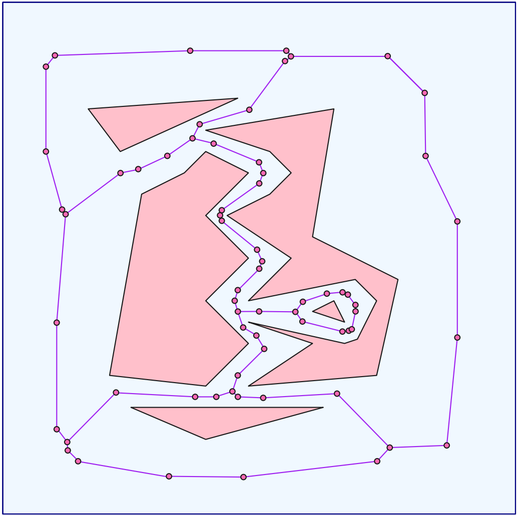
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

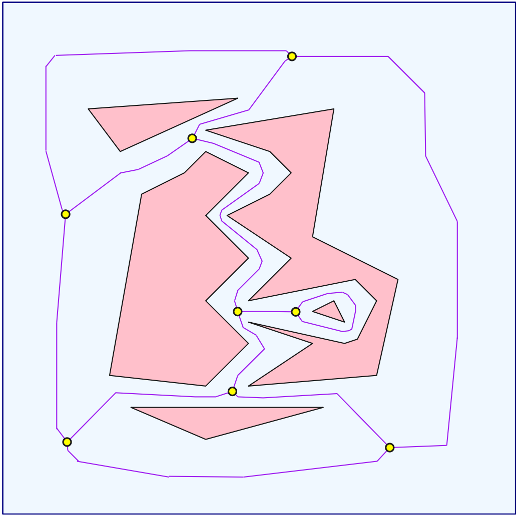
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
   1. Def. by 4 polygons.
   2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

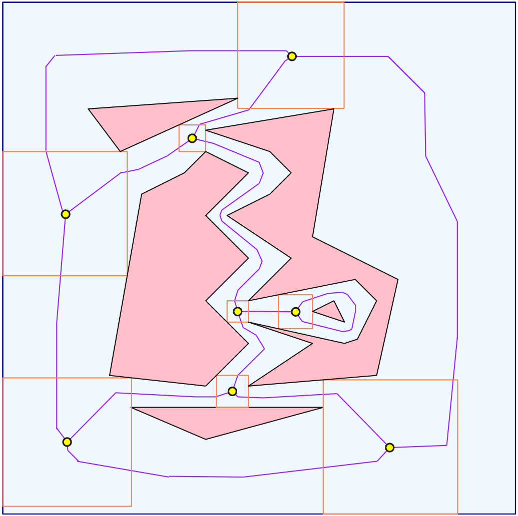
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
   1. Def. by 4 polygons.
   2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

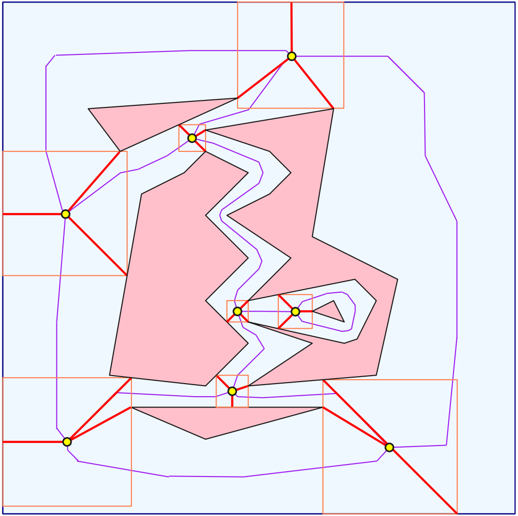
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
   1. Def. by 4 polygons.
   2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

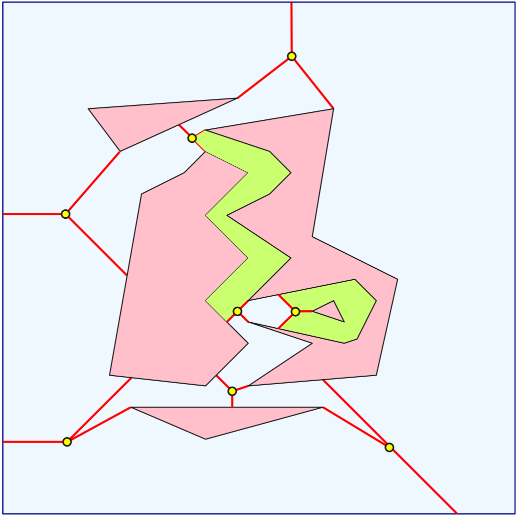
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

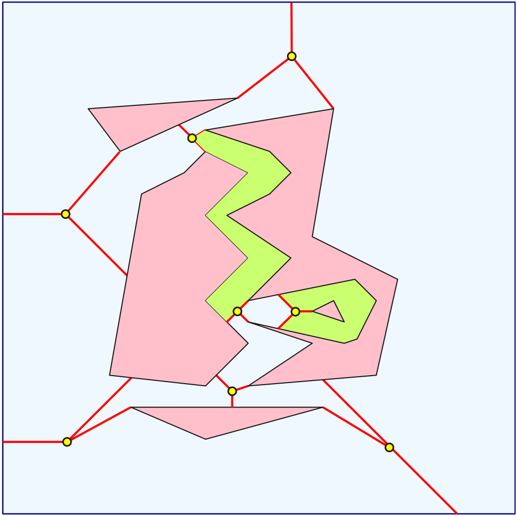
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
   1. Def. by 4 polygons.
   2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

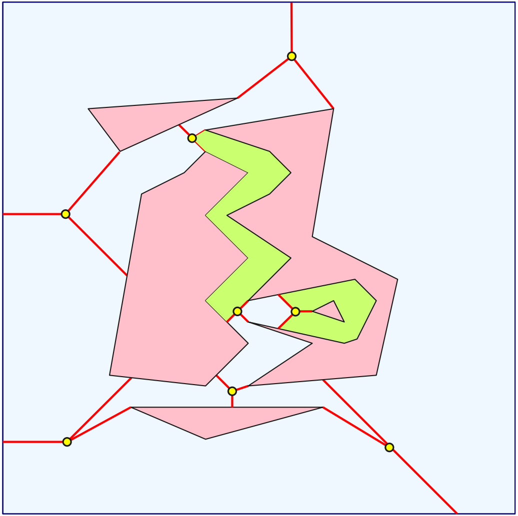
Breaking into cells for polygons



1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

14: Corridor decomposition

Breaking into cells for polygons



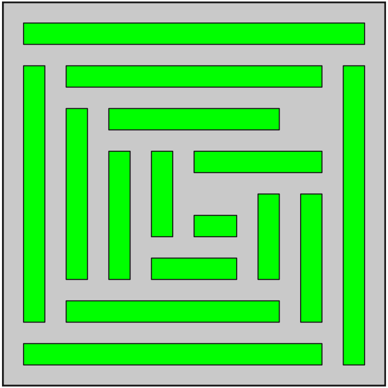
1. L_∞ medial axis. Deformation retract.
2. Critical squares.
3. Tendrils...
4. ...remove them
5. deg 3 Vertices...
6. ...critical squares
7. ...add spokes.
8. ...broke comp. into **corridors**.
    1. Def. by 4 polygons.
    2. $O(m)$ corridors. m: # polygons.

Part V

Cuttings

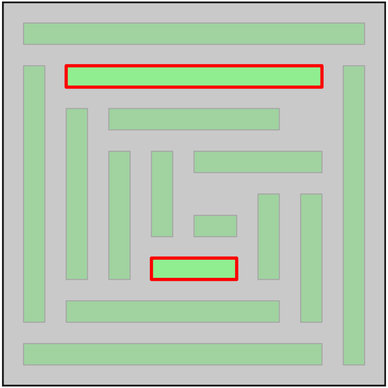
Deep in the technicalities zone

15: Cuttings



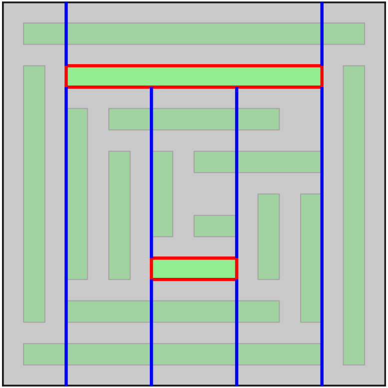
1. $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
   r: Parameter
2. **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
3. **[Chazelle and Friedman, 1990]**
   $1/r$-cutting with $O(r)$ cells.

15: Cuttings



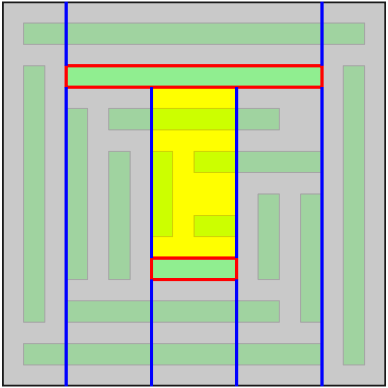
1. $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
   r: Parameter
2. **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
3. **[Chazelle and Friedman, 1990]**
   $1/r$-cutting with $O(r)$ cells.

15: Cuttings



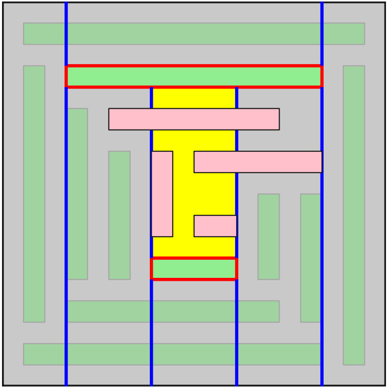
1. $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
   r: Parameter
2. **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
3. **[Chazelle and Friedman, 1990]**
   $1/r$-cutting with $O(r)$ cells.

15: Cuttings



1. $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
   r: Parameter
2. **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
3. **[Chazelle and Friedman, 1990]**
   $1/r$-cutting with $O(r)$ cells.

15: Cuttings



1. $\mathcal{O}$: Set of n disjoint axis-parallel rectangles.
   r: Parameter
2. **$1/r$-cutting**: Partition into cells, s.t. each cell intersects $\leq n/r$ rects $\mathcal{O}$.
3. **[Chazelle and Friedman, 1990]**
   $1/r$-cutting with $O(r)$ cells.

16: Weak cuttings for polygons

Lemma

$\mathcal{P}$: disjoint weighted polygons
W: total weight W
$\implies$ *compute $1/r$-cutting with $O(r \log r)$ corridors.*

1. Proof is extension of previous proof.
2. Might be slightly simpler.

16: Weak cuttings for polygons

Lemma

$\mathcal{P}$: disjoint weighted polygons
W: total weight W
$\implies$ *compute $1/r$-cutting with $O(r \log r)$ corridors.*

1. Proof is extension of previous proof.
2. Might be slightly simpler.

16: Weak cuttings for polygons

Lemma

$\mathcal{P}$: disjoint weighted polygons
W: total weight W
$\implies$ *compute $1/r$-cutting with $O(r \log r)$ corridors.*

1. Proof is extension of previous proof.
2. Might be slightly simpler.

Part VI

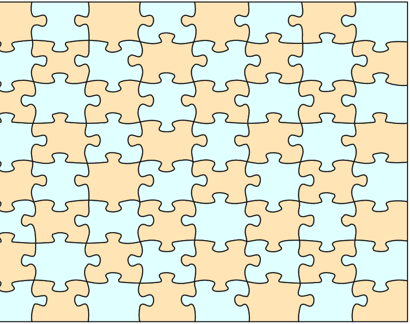
The Approximation Algorithm

A monstrous dynamic programming in action

17: Recursing on optimal solution

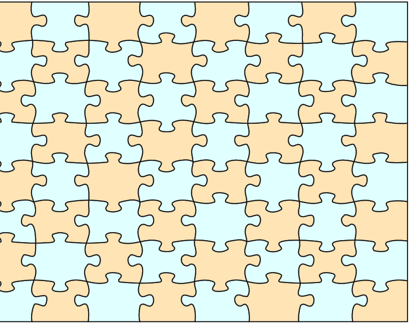
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



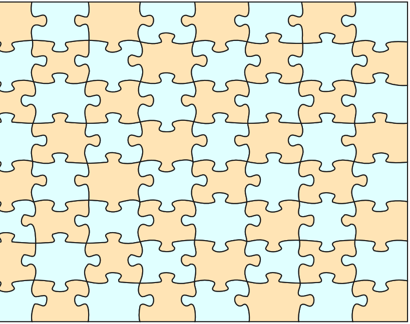
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



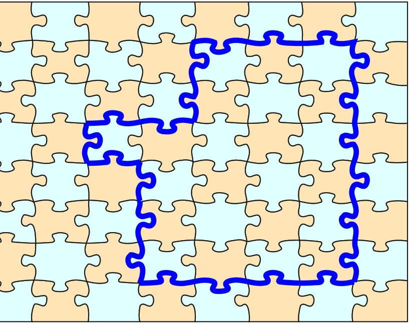
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



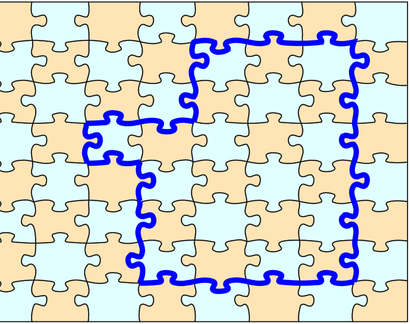
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



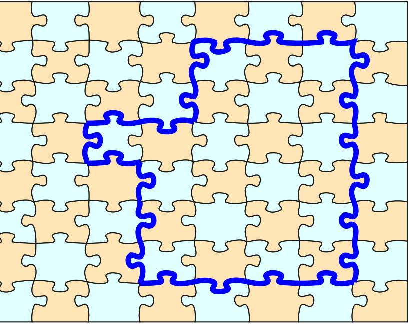
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



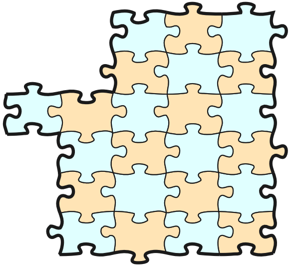
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution



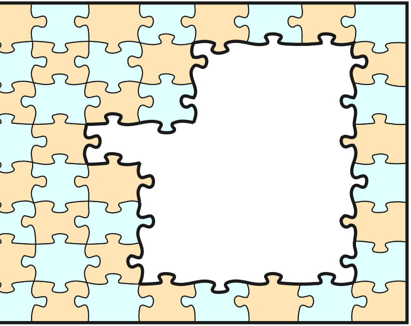
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

 Recursing on optimal solution



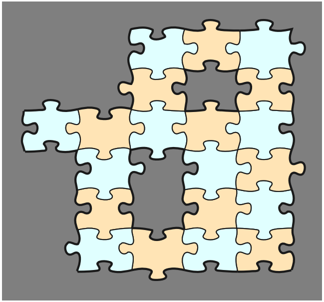
1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

17: Recursing on optimal solution

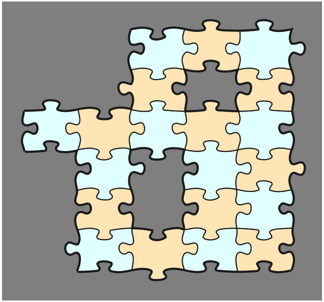


1. $\mathcal{O}$: Opt sol $m = |\mathcal{O}|$, r.
2. $1/r$-cutting, with $\widetilde{O}(r)$ cells.
3. Tiles form a planar graph.
4. tile intersects $\leq \frac{m}{r}$ polys $\in \mathcal{O}$.
5. Cycle separator $\widetilde{O}(\sqrt{r})$ edges.
6. Separator intersects $\widetilde{O}((m/r)\sqrt{r}) = \widetilde{O}(m/\sqrt{r})$.
7. Recurse: In : Out

18: How a subproblem looks like

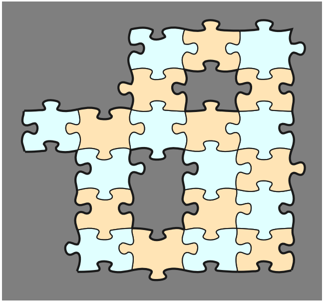


1. $\mathcal{O}$: Opt sol $|\mathcal{O}| \leq m$.
2. Subproblem defined by $O(\log m$
   cycles.
3. Each cycle has $\widetilde{O}(\sqrt{r})$ "corridor
   edges.



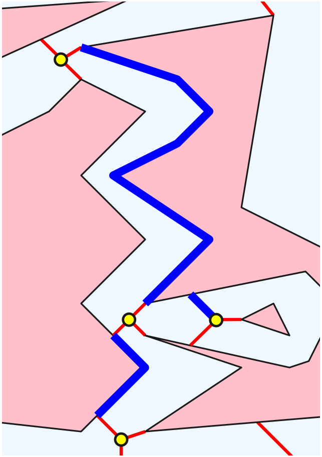
1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.

19: Guessing the separator



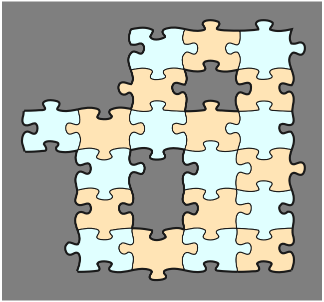
1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.

19: Guessing the separator



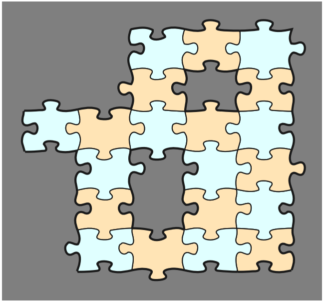
1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.

19: Guessing the separator



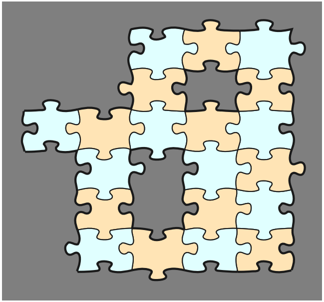
1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.

19: Guessing the separator

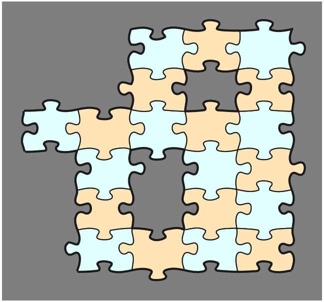


1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r} \log m)}$ different subproblen overall.

19: Guessing the separator



1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.



1. $\mathcal{O}$: Optimal solution
   r: Param (specified shortly).
2. $m = |\mathcal{P}|$.
3. $m^{O(1)}$ possible edges.
   (cuttings+corridors decompositio
4. Separator has $\widetilde{O}(\sqrt{r})$ edges.
5. $m^{\widetilde{O}(\sqrt{r})}$ different subproblems in the root.
6. Depth of recursion $O(\log m)$.
7. $m^{\widetilde{O}(\sqrt{r}\log m)}$ different subproblen overall.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1-\varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r} \log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1-\varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r} \log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1 - \varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}\left(\sqrt{r} \log m\right)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1 - \varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r} \log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1 - \varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r} \log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1 - \varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r}\log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

20: What we lose in the recursion

1. $|\mathcal{O}| \leq m$.
2. Every level...
   Lose $c\sqrt{\frac{\log r}{r}}$ fraction opt sol.
3. $\leq \log m$: depth recursion.
4. Total loss: $c\sqrt{\frac{\log r}{r}} \log m \leq \varepsilon$.
5. $r = O\left(\frac{\log^3 m}{\varepsilon^2}\right) \implies (1 - \varepsilon)$-approximation.
6. $m = |\mathcal{P}|$.
7. Running time $m^{\widetilde{O}(\sqrt{r} \log m)}$.
   $\implies$ Running time $m^{O\left(\left(\frac{\log m}{\varepsilon}\right)^3\right)}$ **QPTAS**.

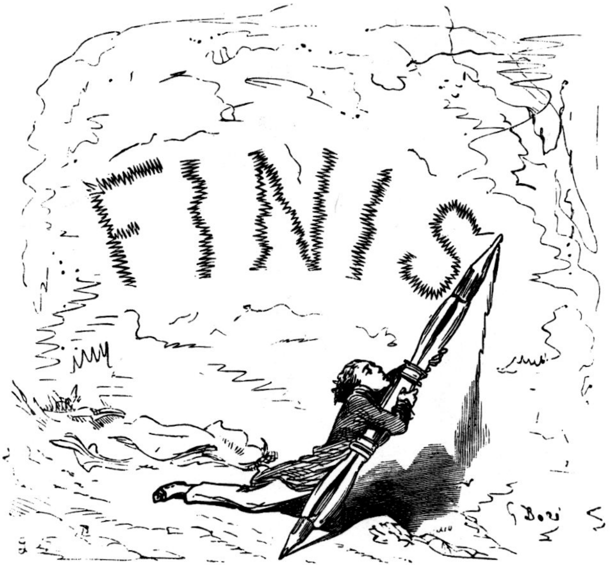
Part VII

Conclusions

This too shell pass

21: Conclusions

1. Framework of **[Adamaszek and Wiese, 2013, 2014]**.
   Insight: Using planar separator on cuttings.
2. In this paper:
   1. Extended framework...
   2. Showed **QPTAS** for independent set of polygons of arbitrary complexity.
   3. Showed techniques works for computing sparse subsets.
3. Open questions for future research:
   1. **PTAS**?
   2. Better running time.


FINIS

A. Adamaszek and A. Wiese. Approximation schemes for maximum weight independent set of rectangles. In *Proc. 54th Annu. IEEE Sympos. Found. Comput. Sci.* (FOCS), pages 400–409, 2013.

A. Adamaszek and A. Wiese. A QPTAS for maximum weight indepdent set of polygons with polylogarithmic many vertices. In *Proc. 25th ACM-SIAM Sympos. Discrete Algs.* (SODA), pages 400–409, 2014.

P. K. Agarwal and N. H. Mustafa. Independent set of intersection graphs of convex objects in 2D. *Comput. Geom. Theory Appl.*, 34 (2):83–95, 2006.

P. Chalermsook and J. Chuzhoy. Maximum independent set of rectangles. In *Proc. 20th ACM-SIAM Sympos. Discrete Algs.* (SODA), pages 892–901, 2009.

T. M. Chan. Polynomial-time approximation schemes for packing and piercing fat objects. *J. Algorithms*, 46(2):178–189, 2003.

T. M. Chan and S. Har-Peled. Approximation algorithms for maximum independent set of pseudo-disks. *Discrete Comput. Geom.*, 48:373–392, 2012.

B. Chazelle and J. Friedman. A deterministic view of random sampling and its use in geometry. *Combinatorica*, 10(3):229–249, 1990.

T. Erlebach, K. Jansen, and E. Seidel. Polynomial-time approximation schemes for geometric intersection graphs. *SIAM J. Comput.*, 34(6):1302–1323, 2005.

J. Fox and J. Pach. Separator theorems and turn-type results for planar intersection graphs. *Adv. Math.*, 219:1070–1080, 2008.

J. Fox and J. Pach. Computing the independence number of intersection graphs. In *Proc. 22nd ACM-SIAM Sympos. Discrete Algs.* (SODA), pages 1161–1165, 2011.

S. Har-Peled. A simple proof of the existence of a planar separator. *CoRR*, abs/1105.0103, 2013.

J. Hastad. Clique is hard to approximate within $n^{1-\varepsilon}$. *Acta Mathematica*, pages 627–636, 1996.

P. Koebe. Kontaktprobleme der konformen Abbildung. *Ber. Verh. Sächs. Akademie der Wissenschaften Leipzig, Math.-Phys. Klasse*, 88:141–164, 1936.

R. J. Lipton and R. E. Tarjan. A separator theorem for planar graphs. In *Proc. GI Conf. Theoret. Comput. Sci.*, 1977.

G. L. Miller. Finding small simple cycle separators for 2-connected planar graphs. *J. Comput. Sys. Sci.*, 32(3):265–279, 1986.

G. L. Miller, S. H. Teng, W. P. Thurston, and S. A. Vavasis. Separators for sphere-packings and nearest neighbor graphs. *J. Assoc. Comput. Mach.*, 44(1):1–29, 1997.