Algorithms & Theory Qual Spring 2019

Part II: Automata and Complexity

March 5, 2019, Tuesday 9am-5pm, SC 4405

1 It is a long way to get accepted.

1.A. [**30 Points**] Given k DFAs $M_1, \ldots, M_k$ each of at most n states, show that if the intersection of their k corresponding languages $L = L(M_1) \cap \cdots \cap L(M_k)$ is nonempty, then L must contain a string of length at most n^k.

1.B. Prove that the following problem is in **PSPACE**: given an integer k and k DFAs $M_1, \ldots, M_k$, decide whether $L(M_1) \cap \cdots \cap L(M_k)$ is empty.

[Note: k is not a constant! Hint: Savitch's theorem, **PSPACE** = **NPSPACE**, may be useful.]

2 A language $S \subseteq \{0,1\}^*$ is *sparse* if there is some polynomial $p(n)$ such that for all $n \geq 0$ that $S \cap \{0,1\}^n \leq p(n)$. For a language L, show that $L \in \mathsf{P}/\text{poly}$ iff $L \in \mathsf{P}^S$ for some sparse language S.

3 Recall the model of interactive proofs, where a computationally unbounded prover P interacts with a polynomial-time probabilistic verifier V. Consider the complexity class of languages with efficient interactive proofs where we consider different completeness and soundness parameters. That is, define $\mathsf{IP}_{c,s}$ to be those languages L where

$$x \in L \implies \exists P \Pr_V[(P \leftrightarrow V)(x) \text{ accepts}] \geq c$$
$$x \notin L \implies \forall P \Pr_V[(P \leftrightarrow V)(x) \text{ accepts}] \leq s.$$

The standard definition of the complexity class of interactive proofs is given by $\mathsf{IP} := \mathsf{IP}_{\frac{2}{3},\frac{1}{3}}$.

3.A. Give a direct proof that $\mathsf{IP}_{\frac{2}{3},\frac{1}{3}} = \mathsf{IP}_{1-\frac{1}{2^n},\frac{1}{2^n}}$, where $n = |x|$.

3.B. Prove that $\mathsf{IP}_{\frac{2}{3},\frac{1}{3}} = \mathsf{IP}_{1,\frac{1}{3}}$.

3.C. Prove that $\mathsf{IP}_{\frac{1}{2}+\frac{1}{2^n},\frac{1}{2}} = \mathsf{IP}_{\frac{2}{3},\frac{1}{3}}$.

4 $k\mathsf{UP}$ are problems solvable in NP by a machine that has at most k accepting computations on any input. We will denote $1\mathsf{UP}$ as simply UP. It is clear that $\mathsf{P} \subseteq k\mathsf{UP} \subseteq (k+1)\mathsf{UP}$ for each k, but not known if the hierarchy is strict. Prove that, $\mathsf{P} = \mathsf{UP}$ iff $\mathsf{P} = 2\mathsf{UP}$.

Aside: This observation can be coupled with induction to conclude that $\mathsf{P} = k\mathsf{UP}$ for some k iff $\mathsf{P} = k\mathsf{UP}$ for all k. Please *do not* prove this generalization.